CHAPTER 14

The Basics of Engineering Mechanics

INTRODUCTION

Engineering mechanics is an important analytical tool that allows an engineer to optimize a design, creating one that it is strong and rigid enough to do the job, but not overly heavy and expensive. A thorough study of mechanics requires some knowledge of calculus and vector mechanics that many students do not ordinarily complete until the end of their sophomore year. Such a course is an essential component of many engineering disciplines, including mechanical, aerospace and civil engineering. The purpose of this chapter is to provide introductory engineering students with some important, basic analysis tools that can be applied to many design projects.

Virtually any physical device in the "real world" is acted on by forces, which can include gravitational, pressure, magnetic, electrostatic, centrifugal and impact forces. Engineering mechanics studies the effects that forces have on materials, allowing engineers to theorize how a design will react before it is built. Engineers may then optimize designs on paper (or with a computer), without having to build and test multiple versions of a product. Using mechanics, better products can be designed and built faster and more cost effectively.

This chapter has four major topics: In *Statics*, the first step in analyzing many physical structures, assumptions are made that all bodies are completely rigid and that they are strong enough to withstand the forces applied. *Mechanics of materials* considers how forces act upon actual bodies in bending, twisting or breaking them (of course, the goal of the designer is to create designs that do *not* bend too much or break). Engineers use various *design criteria* to create physical products, including design for strength, factor of safety and design for stiffness. Finally, *units* that are commonly used in mechanics are presented.

STATICS

Scalars and Vectors

One characteristic of engineers is that they quantify the world around them. *Scalars* are quantities that can be expressed solely in terms of magnitude, such as:

- Area
- Length
- Mass
- Moment of inertia
- Energy
- Power
- Volume
- Work

However, for some quantities, it is necessary to know both a magnitude *and* a direction to describe them completely. These quantities are known as *vectors.* Examples of vector quantities include:

- Force
- Moment
- Momentum
- Displacement
- Velocity
- Acceleration

While scalar quantities can be combined arithmetically, vectors require geometric and/or trigonometric manipulation. Vectors are usually depicted as arrows in a three-dimensional space. As shown in Figure 14.1, the magnitude of a vector is proportional to its length, and the direction of the arrow gives its orientation.

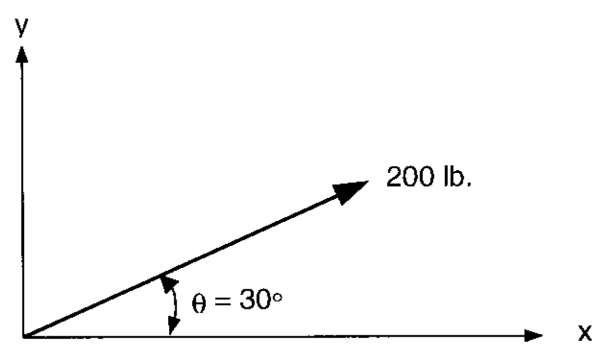


Figure 14.1. Representation of a 200-lb force at an angle of 30 ° with the x-axis.

Vectors can be added or subtracted geometrically and/or trigonometrically. For example, consider two force vectors **A** and **B** that are added together to give the vector sum **C** = **A** + **B** (Figure 14.2)[1]. Geometrically, vector **B** is moved so that its tail coincides with the tip of vector **A**, while retaining its magnitude and direction. A line from the tail of vector **A** to the tip of vector **B** gives the vector sum **C**, also known as the *resultant.*

1. Quantities printed in **bold** are vector quantities, while scalar quantities are printed in normal font.

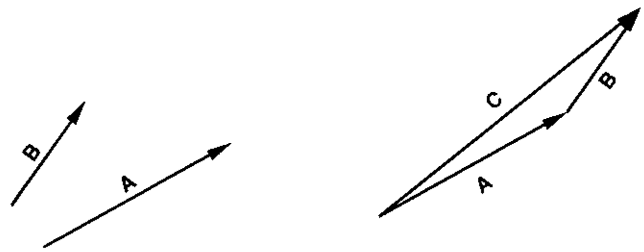


Figure 14.2. The geometric interpretation of the vector sum **C** = **A** + **B**.

By moving vector **A** instead of vector **B**, it becomes evident that one important property of vector addition is that the order is immaterial; i.e., **C** = **(A + B)** = **(B + A)**.

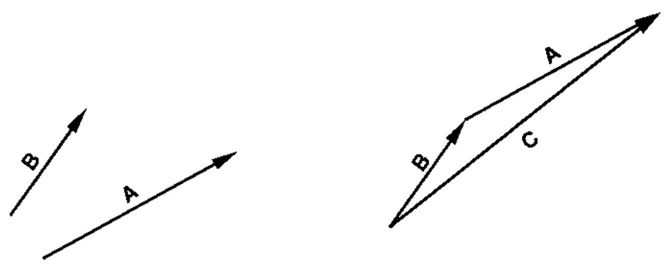


Figure 14.3. The geometric interpretation of the vector sum **C** = **B** + **A**.

Mathematically, the simplest way to manipulate vectors is to utilize a Cartesian[2] coordinate system, which consists of three mutually orthogonal axes, usually designated x, y and z. A vector can then be resolved into three components that lie along those three axes:

$$\mathbf{F} = F_x\,\mathbf{i} + F_y\,\mathbf{j} + F_z\,\mathbf{k}$$

where F_x, F_y and F_z are the Cartesian components (scalar quantities) of the force vector **F**. They are computed by:

$$F_x = F\cos(\theta_x)$$

$$F_y = F\cos(\theta_y)$$

$$F_z = F\cos(\theta_z)$$

where θ_x = angle that **F** makes with the x-axis, etc.

Note that the magnitude (F), of the force vector **F**, can be expressed in terms of its Cartesian components by:

$$F = [F_x^2 + F_y^2 + F_z^2]^{1/2}$$

2. Named after René Descartes (1596-1650), a French mathematician regarded as "the father of modern philosophy," who established the philosophical movement called "rationalism." His theories swept aside the metaphysics of previous philosophers, as he believed the mind possessed a "clear and distinct" idea of self. As he said, "I think, therefore I am" and "It is not enough to have a good mind. The main thing is to use it well."

The unit vectors **i**, **j** and **k** each have a magnitude equal to one, with directions aligned with the x-, y- and z-axes, respectively (Figure 14.4).

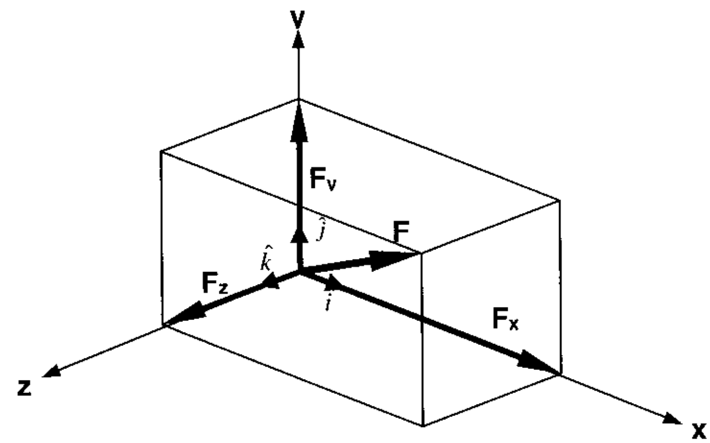


Figure 14.4. Force vector **F** and its Cartesian components.

Adding vectors then becomes a matter of simply adding up all the x, y and z components arithmetically.

Example 14.1

Consider the vector sum **C** = **A** + **B** shown earlier. If we know that vectors **A** and **B** have magnitudes and directions as shown in Figure 14.5, we can resolve them into their Cartesian coordinates:

A= (20) cos(30°)**i** + (20) cos(60°)**j** = 17.32**i** + 10.00**j**

B= (15) cos(50°)**i** + (15) cos(40°)**j** = 10.64**i** + 11.49**j**

The resultant sum **C** can be computed by summing up the x and y components of **A** and **B**:

$$C_x = 17.32 + 10.64 = 26.96$$

$$C_y = 10.00 + 11.49 = 21.49$$

Vector **C** can be combined back into a single vector:

$$C = [C_x^2 + C_y^2]^{1/2} = [(26.96)^2 + (21.49)^2]^{1/2} = 34.48$$

$$\theta_x = \tan^{-1}(C_y/C_x) = \tan^{-1}(21.49/26.96) = 38.56°$$

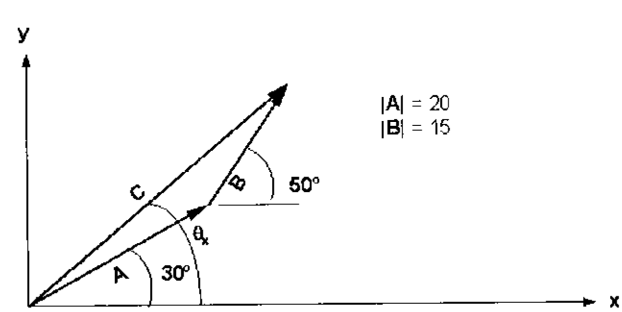


Figure 14.5. Vector sum **C** = **A** + **B**.

Forces

As mentioned above, forces are vector quantities; i.e., it is necessary to know both their magnitude and direction. Some forces develop under static conditions, such as gravitational, magnetic, pressure and electrostatic. Other forces, such as centrifugal and impact forces, develop because of a dynamic condition.

One important property of force vectors is that they can be moved anywhere along their line of action. For example, the rigid body shown in Figure 14.6 is in equilibrium regardless of whether the force, $\mathbf{F_1}$, is pushing (a) or shifted along the line of action so that it is pulling (b).

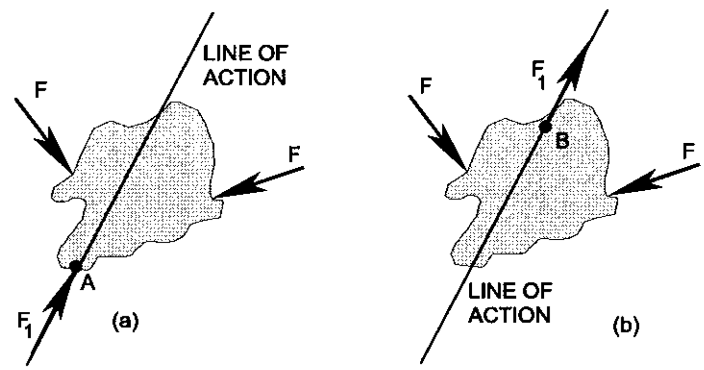


Figure 14.6. A rigid body remains in equilibrium as force $\mathbf{F_1}$ is shifted along its line of action from point A to point B.

Forces may be *external forces* that are applied to a structure, or *internal forces* that develop inside the structure. External forces include active forces applied to a structure as well as the reaction forces that develop at the supports. In Figure 14.7, a simply supported beam is loaded with a concentrated load (a). In (b), the beam is loaded by a uniformly distributed load spread over the top of the beam, or the weight of the beam itself loads the beam. In (c), the supports have been replaced by the reaction forces, $\mathbf{R_1}$ and $\mathbf{R_2}$, which must be present at the supports to maintain equilibrium; this is known as a *free-body diagram*, which is an important tool for visualizing the complete loading state of a structure.

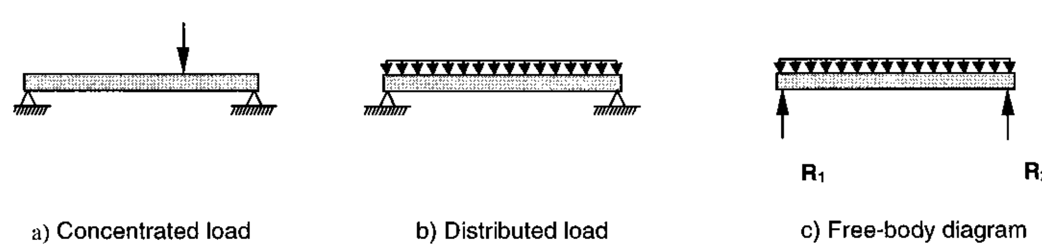
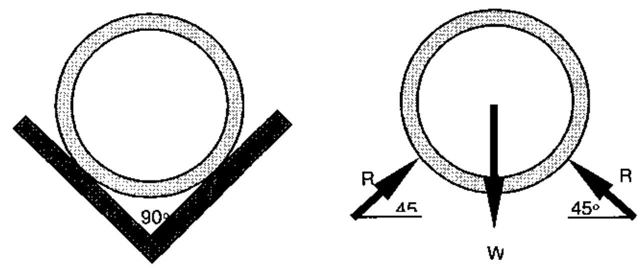


Figure 14.7. External forces applied to a simply supported beam.

Example 14.2



Consider a circular pipe supported in a right-angled rack as shown below. In the free-body diagram on the right, the rack is removed and replaced by the reaction forces **R**, each of which acts at 45° to the horizontal. (It can also be shown that the reaction forces must pass through the center of the circle, which provides a convenient way to find the center of circular objects.)

Internal forces develop within a structure due to the action of the external applied and reaction forces. Although these internal forces are invisible, they can be visualized by making imaginary cuts through a member and solving the equations of equilibrium. For example (Figure 14.8), consider a weight being supported by a rope (a). In the free-body diagram (b), the weight is replaced by a force vector, **W**, acting downward, and the reaction force, $\mathbf{R_1}$ acting upward. By making an imaginary cut through the rope, the internal force, $\mathbf{P_{int}}$, can be found, and can be seen to be equal in magnitude to **W** but opposite in direction.

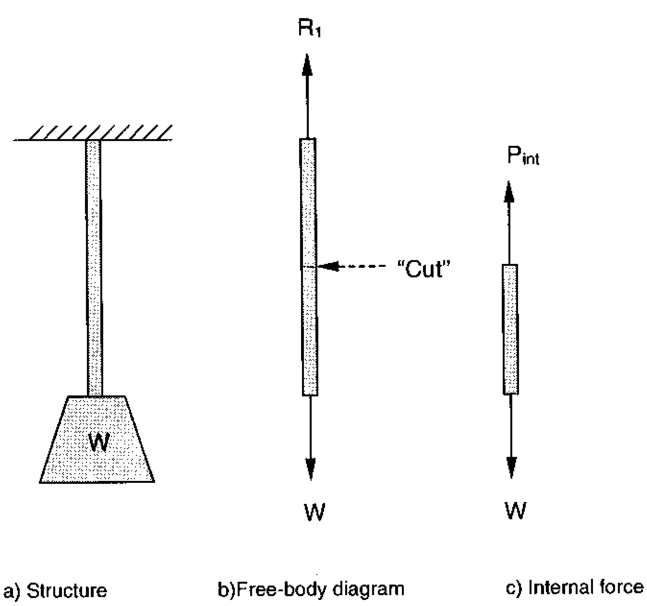


Figure 14.8. Internal forces are computed by making an imaginary cut in a member.

Moments

A moment is the result of a force acting offset from a point. As shown in Figure 14.9, the force **F** tends to make the body rotate clockwise about point **O**. The magnitude of the moment created by this situation can calculated by:

$$M_o = Fd$$

where d = the perpendicular distance to the line of action of **F**.

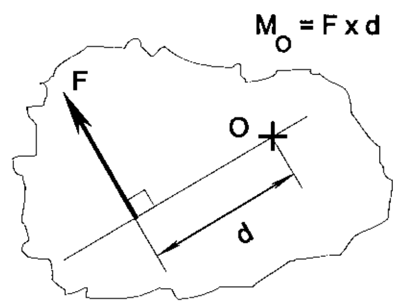


Figure 14.9. The moment produced by a force depends on the position of point **O**.

The units of moment are given as N-m in the SI system or lb-ft in the US system. Incidentally, a moment that tends to bend a bar in its plane of symmetry s referred to as a *bending moment*, and one that tends to twist the bar about its long axis is usually termed as *torsion* (Figure 14.10).

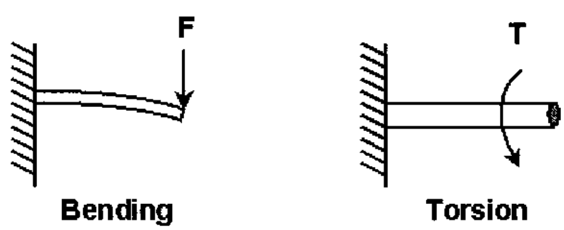


Figure 14.10. Bending moment and torsion.

Equilibrium

The basic principle used in statics to solve for all of the forces and moments acting on a rigid body is based on Newton's[3] First Law, which states that if the sum of all forces acting on a body is zero (i.e., Σ **F** = 0), the body will either remain at rest or continue moving with constant velocity. This condition is referred to as *equilibrium.*

External Forces

Unless it is accelerating or has fractured, a structure with applied external forces is in a state of equilibrium, which means that it must satisfy the equations of equilibrium:

3. Sir Isaac Newton (1642–1727) developed the branch of mathematics known as calculus when he was only 24. He later turned his attention to planetary motion, formulating his famous three laws of motion. Engineering mechanics is commonly referred to as Newtonian Mechanics.

$$\Sigma \mathbf{F} = 0$$

$$\Sigma \mathbf{M} = 0$$

These equations are given in vector format, but it is usually more convenient to solve statics problems by representing the forces and moments in Cartesian components:

$$\Sigma F_x = 0, \qquad \Sigma F_y = 0, \qquad \text{and} \qquad \Sigma F_z = 0 \qquad \text{(Equation 14.1)}$$

$$\Sigma M_x = 0, \qquad \Sigma M_y = 0, \qquad \text{and} \qquad \Sigma M_z = 0 \qquad \text{(Equation 14.2)}$$

It is useful to classify the force system that is acting on a structure, then choose the appropriate sets of the above equilibrium equations to solve for any unknown forces. As shown in Table 14.1, different classifications of force systems require different subsets of Equations 14.1 and 14.2.

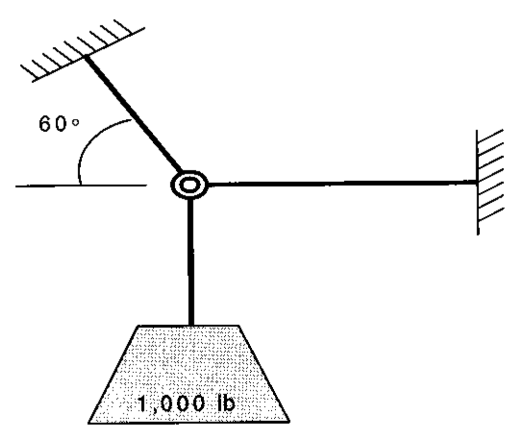
Table 14.1. Classifications of force systems and appropriate equilibrium equations.

Force System Classification	**Equations of Equilibrium**		
Coplanar and concurrent	$\Sigma F_x = 0$	$\Sigma F_y = 0$	
Coplanar and non-concurrent	$\Sigma F_x = 0$	$\Sigma M_z = 0$	$\Sigma F_y = 0$
Non-coplanar and concurrent	$\Sigma F_x = 0$	$\Sigma F_y = 0$	$\Sigma F_z = 0$
Non-coplanar and non-concurrent	$\Sigma F_x = 0$	$\Sigma M_x = 0$	$\Sigma F_y = 0$
	$\Sigma M_y = 0$	$\Sigma F_z = 0$	$\Sigma M_z = 0$

The following example illustrates a simple six-step procedure for solving for the unknown forces in a coplanar, concurrent force system.

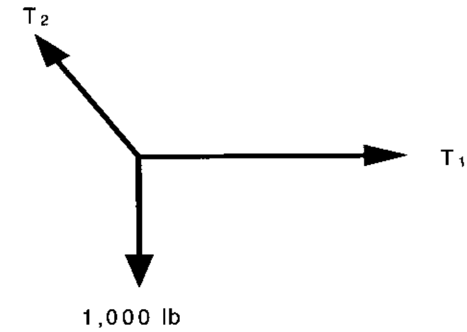
Example 14.3

Consider a 1,000-lb weight suspended from two cables as shown.



Step 1. Prepare a complete free-body diagram of the system as shown above.

Example 14.3 (Continued)



Step 2. Classify the force system. In this problem, the force system is *coplanar* and *concurrent.*

Step 3. Write the applicable equations of equilibrium. In this case, only two of the six equations of equilibrium are applicable.

$\Sigma F_y = 0$ $\Sigma F_x = 0$

Step 4. Substitute the unknowns from the free-body diagram into the applicable equations of equilibrium.

$\Sigma F_y = 0$ $\Sigma F_x = 0$

$T_2 \sin(60^\circ) - 1000 = 0$ $T_1 - T_2 \cos(60^\circ) = 0$

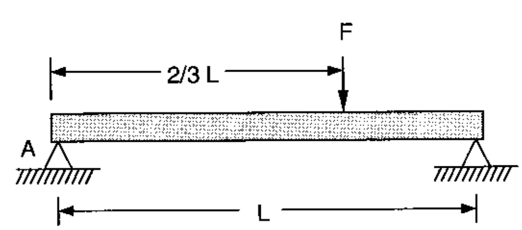
Step 5. Solve the equations.

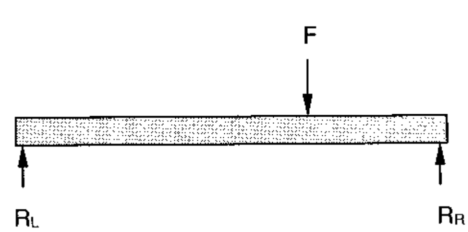
$T_2 = 1000/\sin(60^\circ) = 1155$ lb $T_1 = T_2 \cos(60^\circ) = 577$ lb

Step 6. Check the solution and interpret the results. The simplest way to verify the solution is to resolve T_2 into its x- and y-components and notice that the forces sum to zero in both directions. T_2 should be greater than **W** – is it?

Example 14.4

Consider a simply supported beam with a concentrated load as shown below.





Example 14.4 (Continued)

Step 1. Prepare a complete free-body diagram.

Step 2. Classify the force system, which in this case is *coplanar* and *non-concurrent.*

Step 3. Write the applicable equations of equilibrium (see Table 14.1).

$\Sigma F_x = 0$ $\Sigma F_y = 0$ $\Sigma M_A = 0$

The moment equation can be taken about any arbitrary point. In this case, point A is selected because it eliminates the moment due to R_L, which simplifies the resulting equations.

Step 4. Substitute the unknowns from the free-body diagram into the equations of equilibrium.

$$\Sigma F_y = R_L + R_R - F = 0 \quad \text{(a)}$$

$$\Sigma M_A = R_R L - F(2/3)L = 0 \quad \text{(b)}$$

Step 5. Solve the equations. Because only one unknown is involved, it is simplest to solve Equation (b) first:

$R_R = 2/3\ F$

which is substituted into Equation (a) to give:

$R_L = 1/3\ F$

Step 6. Check the solution and interpret the results. Suppose that L = 15 ft and F = 9000 lb. Substituting these values into the solutions above yields:

$R_R = 2/3\ F = 6000$ lb

$R_L = 1/3\ F = 3000$ lb

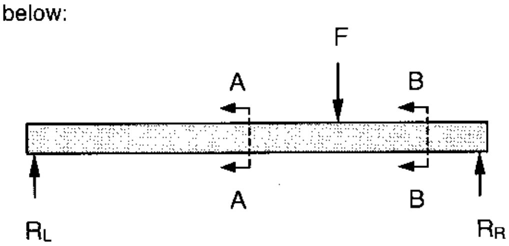
Clearly, $R_R + R_L = F$. In addition, it should seem intuitively logical that R_R is greater than R_L because the force is located at 2/3 L.

Internal Forces and Moments

In order to be able to accurately analyze many structural elements (e.g., beams that are in a state of bending), it is necessary to be able to calculate internal forces and moments. As shown in the following example, this is performed by constructing a series of imaginary slices through the element, and then applying the appropriate equations of equilibrium.

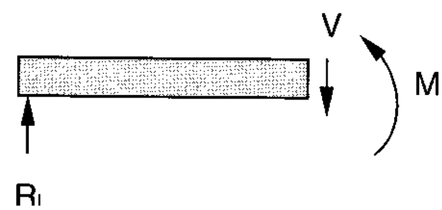
Example 14.5

Calculate the internal forces and moments in the beam analyzed in Example 14.4. The complete free-body diagram, showing all the applied and reaction forces, is shown below:



Example 14.5 (Continued)

An imaginary cut is made at A-A, located a distance x from the left support, but not past the applied load F. The right-hand portion of the beam is removed, and a free-body diagram of the left-hand remainder is drawn:



Since there are no external forces in the x-direction, the only forces will be in the y-direction. In order to satisfy equilibrium, there must be another vertical force to balance the reaction force R_L. The force V (referred to as a *shear force*) is drawn on the right end of the sectioned portion of the beam. Because V and the vertical reaction force, R_L = 3000 lb, are offset by the distance x, they both create a moment. Therefore, there must also be an internal moment, M, to counteract the applied moment. By applying the equilibrium equation:

$$\Sigma F_y = 0 = R_L - V$$

it is evident that $V = R_L = 3000$ lb

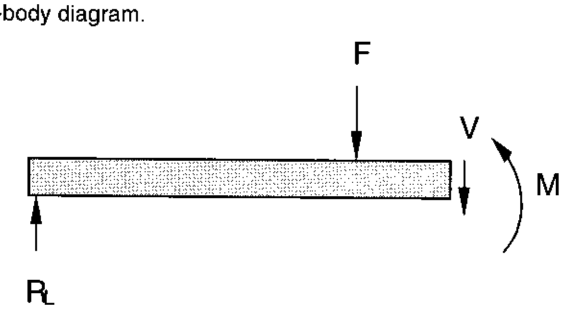
In other words, there is a constant vertical shear force, V, which is equal in magnitude, but opposite in direction to R_L anywhere along the left-hand portion of the beam. By applying the other equilibrium equation:

$$\Sigma M_{A\text{-}A} = 0 = -R_L\, x + M$$

It can be seen that the internal bending moment increases linearly with x:

$$M = R_L\, x \quad 0 < x < 10 \text{ ft}$$

A similar imaginary cut along section B-B, to the right of the applied load, yields a similar free-body diagram.



Applying the first equilibrium equation:

$$\Sigma F_y = 0 = R_L - F - V$$

$$3000 - 9000 - V = 0$$

which leads to $V = -6000$ lb

The negative sign indicates that the assumption that V points downward is incorrect, and in fact, V is an upward shear force in this portion of the beam.

Example 14.5 (Continued)

To find the internal bending moment in this portion of the beam, again, write a moment equation:

$$\Sigma M_{A\text{-}A} = 0 = -R_L\, x + F(x\text{-}10) + M$$

$$-3000x + 9000(x\text{-}10) + M = 0$$

which leads to $M = 90000 - 6000x \qquad 10 < x < 15$ ft

These results can be summarized graphically in shear and bending moment diagrams (Figure 14.11). The main point to notice is that the bending moment is maximum under the applied load and is zero at the supports.

Shear and bending moment diagrams for several common loading conditions are shown in Appendix 13.A, located at the end of this chapter. Detailed equations for deformation and stress of more complex structures and/or loading situations may be found in other references [1].

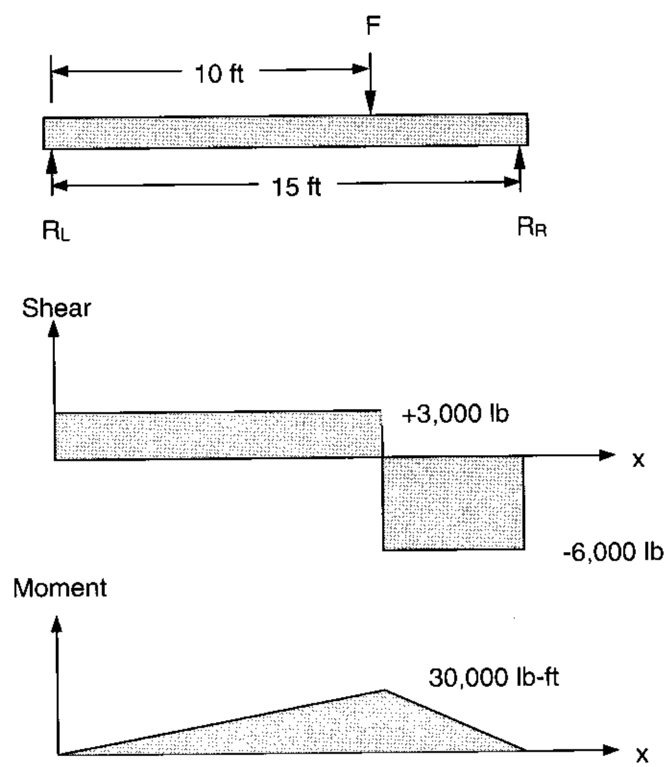


Figure 14.11.Shear and bending moment diagrams for a simply supported beam.

MECHANICS OF MATERIALS

In analyzing problems from a statics viewpoint, structures are considered to be ideal rigid bodies that neither deform nor fail. Statics is a useful tool that allows all the external and internal forces to be calculated. In reality, or course, structures deform and fail, depending on the material they are made from and the loads applied. In order to analyze how materials actually behave under load, it is necessary to introduce the concepts of stress and strain. In order to analyze structures from this point of view, it is necessary to first use statics to solve for all the external and internal forces that act on a body.

Stress

Internal forces that develop within a structural member when it is subjected to external loads generate stresses in the material. Although the state of stress in most real structures is complex and beyond the scope of this text, some simple concepts are useful tools for analyzing many engineering designs.

Tension

The simplest state of stress arises in uniaxial tension (e.g., a rope). In this case, a *normal stress*[4] is uniformly distributed across the circular cross section of the rope (Figure 14.12). Normal stress is calculated by:

$$\sigma = P/A \qquad \text{(Equation 14.3)}$$

where P = internal force at any point [lb] or [N.]

A = cross-sectional area [in.^{2}] or [mm^2]

The units of stress are usually pounds per square inch (psi), or Newtons per square millimeter, more commonly called megapascal (MPa)[5].

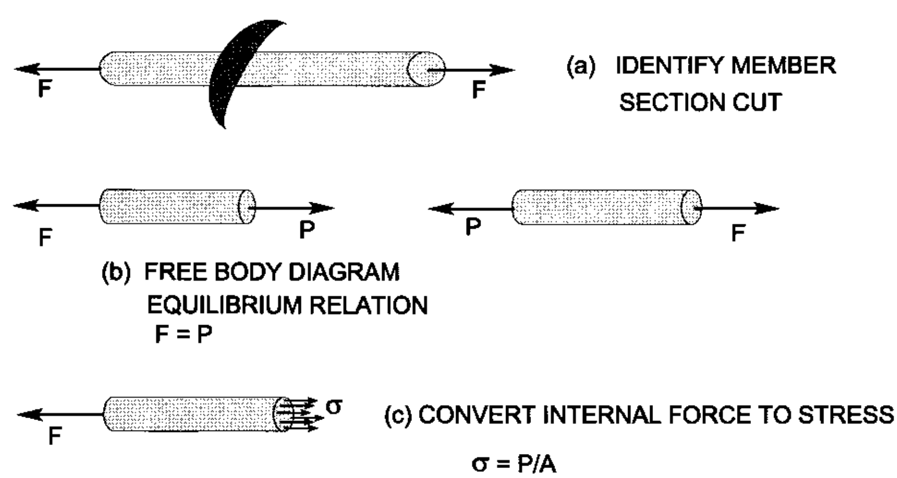


Figure 14.12.Normal stress due to uniaxial tension.

4. "Normal" means perpendicular to the circular cross-section, as opposed to "usual."
5. Blaise Pascal (1623-1662), the French thinker and mathematician worked intensely on scientific and mathematical questions during his short life. He invented a mechanical calculator to help his father, a tax collector. He became interested in probability while calculating odds when gambling. The Pascal programming language is named after him, and the unit, a Pascal, is one Newton per square meter.

Compression

Uniaxial compression is opposite to tension; i.e., a material is being squeezed together instead of pulled apart. The normal stress for this type of loading is also calculated by Equation 14.3. However, engineers should be aware that any member in compression may suddenly buckle laterally, especially if it is long and slender (Figure 14.13). A more complete reference should be consulted if buckling is suspected [2].

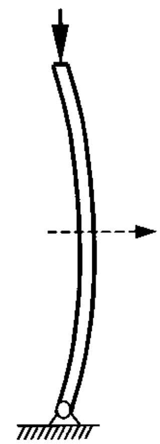


Figure 14.13. Long, slender members in compression may buckle laterally unexpectedly.

> **Example 14.6**
>
> Calculate the compressive stress underneath a 150-lb woman wearing a "spike" high-heeled shoe with a ¼ in. circular diameter.
>
> The area of a circle is: $A = \pi D^2/4 = \pi (.25)^2/4 = .049 \text{ in.}^2$
>
> The stress is then: $\sigma = P/A = 150 \text{ lb}/.049 \text{ in.}^2 = 3056 \text{ psi}$

Bending

Bending describes a more complex state of loading than pure tension or compression. In general, bodies are weaker and deflect more in bending than in pure tension or compression. For example, a paper clip can be easily straightened out by hand. But once straight, it is impossible to apply enough force by hand to stretch or pull the wire apart in tension. The state of stress for a body in bending is shown in Figure 14.14, which shows that the normal stress varies linearly across the section, from compression on one side to tension on the other.

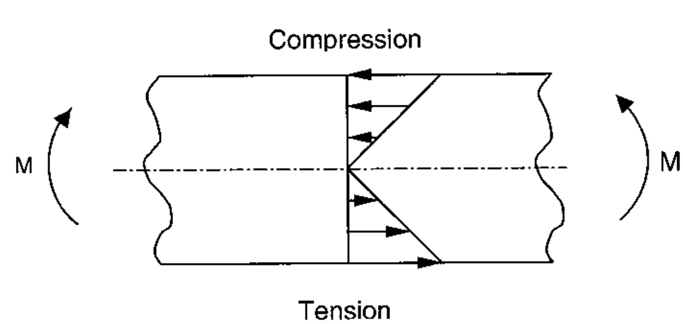


Figure 14.14.State of stress for a body in bending.

Although the stress varies linearly across the section, the designer is primarily interested in the maximum normal stress, which is given by:

$$\sigma_{max} = \pm Mc/I \qquad \text{(Equation 14.4)}$$

where

M = bending moment

I = rectangular moment of inertia

c = half the height of the beam

The plus sign signifies a tensile stress, and the minus sign a compressive stress. The rectangular moment of inertia, I, is a geometric property of the cross section. While the calculation of I for a general shape can be a complex calculation, Table 14.2 lists the formulae for I for several commonly encountered shapes.

Although each state of bending is unique, some general rules apply:

- The maximum stress occurs where the bending moment is maximum. Knowing the maximum bending moment requires a complete understanding of all the forces acting on the body (see Example 14.5).
- The state of stress for a body in bending varies from tension on one side to compression on the other, but almost all engineering analyses are based on maximum stress as given by Equation 14.4.

Table 14.2. Rectangular moment of inertia (I) for some simple shapes.

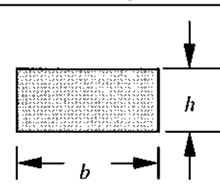
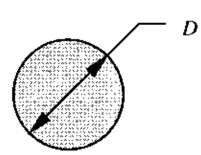
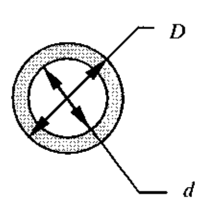
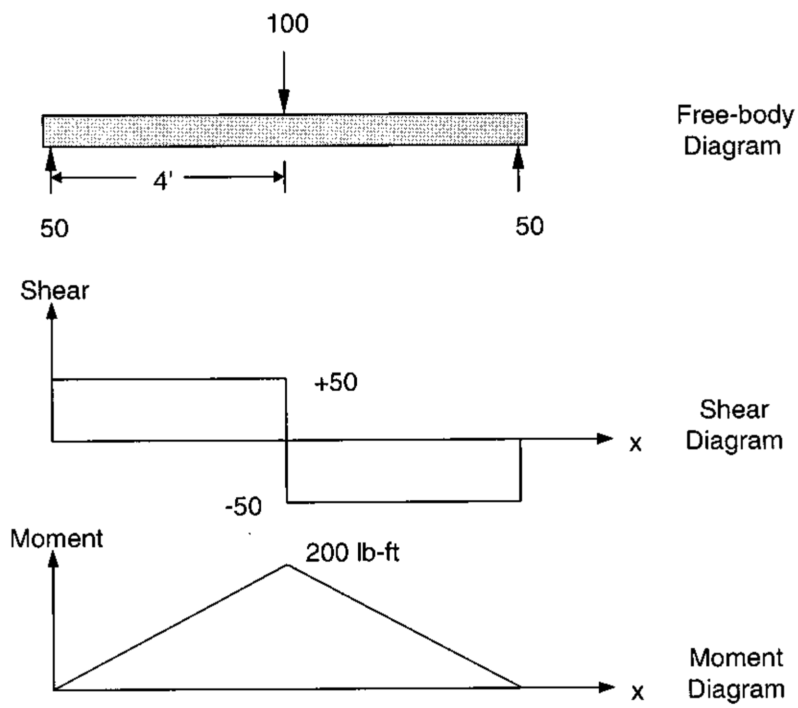
Rectangular 	$I = \frac{bh^3}{12}$
Solid Circular 	$I = \frac{\pi D^4}{64}$
Hollow Circular 	$I = \frac{\pi(D^4 - d^4)}{64}$

Table 10.2 Rectangular moment of inertia (I) for some simple shapes

Example 14.7

Calculate the maximum bending stress for a wooden "2 by 4" eight feet long supporting a 100-lb boy, if it is: a) lying flat, or b) standing on edge.

It is necessary to first compute the maximum bending moment. The free-body diagram shown below leads to the moment diagram, which reveals the maximum moment to be 200 lb-ft.



a) A 2 x 4 actually measures 1.5 in. by 3.5 in.[6] Therefore, the moment of inertia is given by (see Table 14.2):

$I = 1/12\ bh^3 = 1/12\ (3.5)(1.5)^3 = .984\ in.^4$

The maximum normal stress is calculated by Equation 14.4:

$\sigma_{max} = \pm Mc/I = \pm$ (200 lb-ft)(12 in./ft)(.75 in.)/(.984) = $\pm$1830 psi

This value for σ_{max} can be compared to the strength of wood to see if it will break. Douglas fir, a common type of wood, can withstand a stress of approximately 6500 psi before it breaks. This allows us to calculate the factor of safety (described in more detail later):

Factor of Safety (FS) = Strength/Stress, or

FS = 6500/1830 = 4.8

Since FS > 1, we can predict that this board will *not* fail.

6. When lumber was first used for construction, it was rough-sawn into various rectangular shapes, including one that measured 2 in. by 4 in. Later, lumber was given a smooth finish cut, reducing its overall size to 1.5 in. by 3.5 in., but the original name of "2 by 4," or "2 x 4," is still used.

Example 14.7 (Continued)

b) When the board is standing on edge, it is necessary to use different values to calculate I and c:

$I = 1/12\ bh^3 = 1/12\ (1.5)(3.5)^3 = 5.36\ in.^4$

which gives rise to a bending stress of:

$\sigma_{max} = \pm Mc/I = \pm\ (200\ lb\text{-}ft)(12\ in./ft)(1.75\ in.)/(5.36) = \pm 784\ psi$

Notice that the bending stress when the board is standing on edge is less than half what it is when lying flat. Since most of the loads in buildings are due to gravity, boards that support floors ("joists") or roofs ("rafters") are always mounted on edge.

Torsion

Torsion describes the situation when a long, slender component is twisted about its long axis. As shown in Figure 14.15, a circular shaft in torsion develops a *shear stress* (ô) that varies linearly from zero at the center to a maximum at the outer diameter of the shaft.

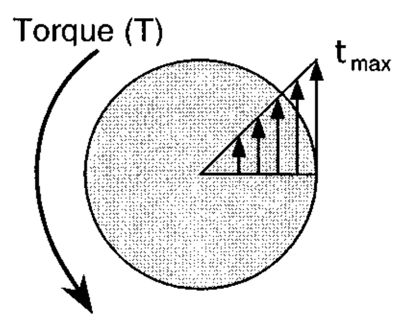


Figure 14.15.Distribution of shear stress in a shaft in torsion.

The maximum shear stress can be calculated by:

$\hat{o}_{max} = \pm Tc/J$ (Equation 14.5a)

where

T = applied torque

J = polar moment of inertia

c = half the diameter of the shaft

Since most shafts are circular, we know that the polar moment of inertia is given by:

J = ðd^4/32

Substituting into Equation 14.5a, we can obtain the following equation for maximum shear stress:

$$\hat{o}_{max} = \frac{16T}{\pi d^3}$$ (Equation 14.5b)

When computing factor of safety, one needs to compare the maximum shear stress with the allowable shear strength for the material. Finding this material property is beyond the scope of this text, but for now, use half the published value for ultimate tensile strength.

Strain

If bodies were perfectly rigid, they would not deflect when loads were applied to them. However, most engineering materials are *elastic,* which means that they deform linearly under load according to Hooke's Law[7]:

$\sigma = E\,\varepsilon$ (Equation 14.6)

where
- σ = normal stress [psi] or [Mpa]
- E = material property known as the modulus of elasticity [psi] or [Mpa]
- ε = strain in the material [in./in.] or [mm/mm]

Tension or Compression

By substituting the definitions of stress and strain into Equation 14.6, the deformation for a body in pure tension or compression (assuming it does not buckle laterally) is given by:

$\delta = PL/AE$ (Equation 14.7)

where
- P = applied load [lb] or [N]
- L = length of wire [in.] or [mm]
- A = cross-sectional area of wire [in.^{2}] or [mm^2]

Torsion

A common engineering component is a circular shaft used to transmit torque. For example, the engine in an automobile delivers torque to the driving wheels through a drive shaft. A shaft in torsion (Figure 14.16) twists through an angle that can be calculated by:

$\theta = TL/JG$ (Equation 14.8a)

where
- θ = angle of twist [rad]
- T = applied torque [lb-in.] or [N-m]
- L = length of shaft [in.] or [mm]
- J = polar moment of inertia ($J = \pi d^4/32$ for circular shafts)
- G = shear modulus of material (see Table 14.3)

Since most shafts *are* in fact circular, we can rewrite Equation 14.8a to be:

$$\theta = \frac{32TL}{\pi d^4 G}$$ (Equation 14.8b)

7. Robert Hooke (1635–1703), one of the greatest experimental scientists of the 17th century, was a generalist of astonishing scientific scope who made lasting contributions to our understanding in optics, mechanics, geography, architecture, materials science, clock-making, paleontology and microbiology. He is less known for his realization—250 years before Darwin—that fossils document the changes to organisms on the planet.

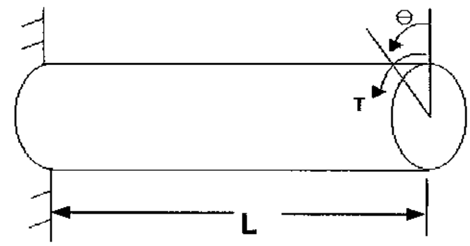


Figure 14.16.Shaft in torsion.

Bending

Beams in bending are more common in most engineering designs than pure tension or compression. Bending deflection is a complex phenomenon that is beyond the scope of this text. However, the equations in Appendix 14.A can be used to calculate many commonly encountered loading conditions.

Table 14.3. Properties of some common materials.

Material	**Modulus of Elasticity** (E) GPa (Mpsi)	**Modulus of Rigidity** (G) GPa (Mpsi)	**Yield Strength** (S_y) MPa (Kpsi)	**Ultimate Strength** (S_{ut}) MPa (Kpsi)
1018 Steel	207.0 (30.0)	79.3 (11.5)	220 (32.0)	341 (49.5)
303 Stainless Steel	190.0 (27.6)	73.1 (10.6)	267 (40.0)	601 (87.3)
2024-T4 Aluminum	71.0 (10.3)	26.2 (3.8)	296 (43.0)	446 (64.8)
Copper	119.0 (17.2)	44.7 (6.5)	69-304 (10-44)	221-331 (32-48)
Wood (Douglas Fir)	11.0 (1.6)	4.1 (0.6)	24.8 (3.6)	45 (6.5)
Plexiglass (Acrylic)	2.6-3.5 (.37-.5)	-	N/A	62-86 (9-12.5)
ABS Plastic (GP)	2.1 (.31)	-	N/A	41 (5.9)
PVC Plastic	2.1-3.5 (.3-.5)	-	N/A	35-55 (5-8)
P1500 Polyester[a]	.83 (.13)	-	N/A	19 (2.8)

a. This material is used in the Genisys rapid prototyping machine.

Example 14.8

Calculate the amount of bending for the 2 x 4 boards from Example 14.7. From Appendix 13.A(c) the deflection for a simply supported beam is given by:

$y = Pl^3/48EI$

a) Therefore, for the board lying flat, the deflection is calculated to be:

$y = (100\text{ lb})(96\text{ in.})^3/(48)(1.6 \times 10^6\text{ psi})(.984\text{ in.}^4) = .293\text{ in.}$

b) When the board is on edge, the deflection is found to be:

$y = (100\text{ lb})(96\text{ in.})^3/(48)(1.6 \times 10^6\text{ psi})(5.36\text{ in.}^4) = .054\text{ in.}$

Notice that the deflection is also considerably less when the board is on edge, as well as the stress.

DESIGN CRITERIA

Design for Strength

As shown earlier, forces acting on a body induce internal stresses. If the stresses become too large, the material can no longer resist them, and it will either permanently deform, or even fracture. Consider a straight length of wire with a tensile force, P, applied to it and then increased. Some materials, like ceramics and many plastics, are classified as *brittle*, which means that they have relatively little deformation and fail abruptly. A good example of brittle failure is a piece of chalk, which does not show any apparent signs of bending, and then suddenly snaps. A stress-strain plot, shown in Figure 14.17, shows that a brittle material will fracture when the stress exceeds the ultimate tensile strength of the material, S_{ut}.

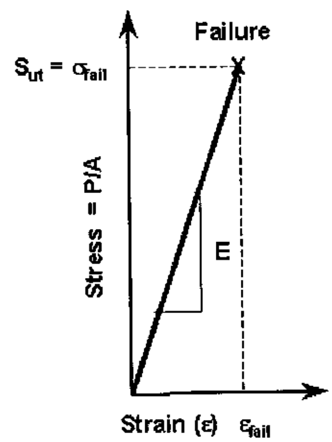


Figure 14.17.Stress-strain graph for a brittle material.

Since tensile stress for pure tension was previously defined to be:

$$\sigma = P/A$$

We can predict that this material will fail when the stress exceeds the strength, or when:

$$P > A\,S_{ut}$$

Strengths for various commonly used materials are given in Table 14.3.

Although bending is characterized by a variation in stress across the section of a part, the same criterion can be used to predict failure; i.e., when the maximum stress, which occurs at the outside "fiber" of the part, exceeds the material's ultimate tensile strength.

Other materials, like most metals, exhibit *ductile* behavior. When the stress becomes sufficiently high, the material begins to *yield*, as shown in Figure 14.18. If the load is relaxed after the material yields, it will be permanently deformed. For example, a paper clip can be bent with enough force, and it will stay bent when the force is removed. The material property that quantifies a material's ability to withstand yielding is termed *yield strength*, S_y (see Table 14.3).

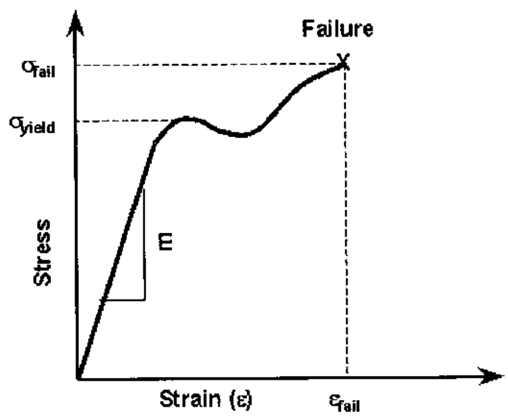


Figure 14.18.Stress-strain plot for a ductile material.

Even if a part has been bent but has not actually fractured, it can no longer do the job for which it was designed. Therefore, yield strength is usually used as the design criterion for strength of ductile materials. Brittle materials, which do not exhibit yielding, are designed using ultimate strength as the design criterion.

Factor of Safety

The equations presented earlier are idealized models that describe material behavior. The material properties (e.g., yield strength) listed in Table 14.3 are typical values for a given material. However, there is always some uncertainty about the actual strength of a material. Similarly, there is always some uncertainty about the actual loads that a part will encounter. For example, assume an engineer has designed a chair to support someone's weight of *exactly* 250 lb. But if a 250-lb person sits down abruptly, and the impact causes the chair to collapse, is it the user's fault, or the engineer's? Or, perhaps the engineer specified a material with a yield strength of 60,000 psi, but one particular batch of steel only had a strength of 57,000 psi. That chair will not support even a static load of 250 lb.

Therefore, *all* designs should include an appropriate factor of safety, which can be calculated by:

$$FS = \text{Material Strength/Applied Stress} = S/\sigma$$

Note that $FS > 1$. An appropriate standard or code may specify the actual factor of safety.

For example, the State of California mandates that the factor of safety for the wire rope that suspends a passenger elevator must be from 7.6 to 11.9, depending on the speed of the elevator [3]. In other words, an elevator should be able to support up to 11.9 times its rated load before it fails. In other cases, it is up to the engineer to decide the appropriate factor of safety to use, which is a function of:

- Degree of certainty in material properties
- Degree of certainty in loading conditions
- Consequence of failure

It may be surprising to know that the structure of an airplane is typically designed with a factor of safety of only 1.5! Although the consequence of a wing falling off is extremely severe, both the loads that an airplane experiences and the materials from which it is made are extremely well known. If a plane were made with the same factor of safety as an elevator, it would be too heavy to get off the ground.

Design for Rigidity

Rigidity is usually a more common design goal than strength. For example, it is possible to design the floor of a house so that it will safely withstand the force of someone walking across it. Yet, if the floor is too springy (i.e., it deflects noticeably when a person walks across it), it will be perceived as unsafe, even though it is actually not. This criterion is usually specified as the maximum amount of deflection that a structure may have.

Example 14.9

Assume a 50-in. rod must support a load of 100 lb and not extend more than .050 in. If an aluminum (E = 10,000,000 psi) rod is chosen, Equation 14.12 can be solved for the required rod diameter, D = .113 in. But, recall that one of the basic design rules is to "use standard sizes," and this diameter rod is not available, at least not at reasonable cost. Therefore, 1/8 in. (.125) should be selected. Equation 14.12 predicts that the actual deflection will be .041 in., which is within the design requirement of .050 in. maximum deflection.

If even less deflection is required, the designer has two options: use a stiffer material (e.g., steel, which has an elastic modulus three times that of aluminum, or use a larger diameter rod).

The engineer should also check the stress in the rod to ensure that it can handle the 100-lb load. Using Equation 14.9, the stress is found to be 8,150 psi, which is well below the yield strength of aluminum given in Table 14.3.

UNITS

In the study of mechanics, there are four basic quantities as shown in Table 14.4 (the gravitation constant, g, relates the units to each other but is not a quantity of measurement in itself).

Table 14.4. Basic quantities and units.

Unit of Measure	US Customary (FPS)	International System of Units (SI)
Length	Foot (ft)	Meter (m)
Time	Second (s)	Second (s)
Mass	Slug ((lb•s^2)/ft)	Kilogram (kg)
Force	Pound (lb)	Newton (N) (kg•m)/s^2
Gravitation Constant g	32.17 ft/s^2	9.807 m/s^2

Two of the units (length and time) are independent. However, in order for force and mass to be dimensionally homogenous, they are related to each other by Newton's second law:

$$\Sigma F = ma$$

In the International System of Units (SI), the units for length, time and mass are specified, and then used to derive the remaining basic unit for force. Length is given in meters (m), time in seconds (s), and mass in kilograms (kg). The unit for force is called a Newton (N) in honor of Sir Isaac. The Newton is derived from Equation 14.16 so that a force of 1N will impart an acceleration of 1 m/s^2 to a mass of 1 kg [i.e., 1 N = (1 kg) (1 m/s^2)]. For dimensional homogeneity, it is clear that N = (kg•m)/ s^2. In the SI system, the gravitation constant g = 9.807 m/s^2. With this value of the acceleration due to gravity on Earth, the weight of a mass of 1 kg is:

$$W = mg = (1\text{kg})(9.807\ \text{m/s}^2) = 9.807\ \text{N}$$

In the US Customary System, length is given in feet (ft), time in seconds (s) and force in pounds (lb). The unit for mass is called a slug, which is derived from Equation 14.16 so that a force of 1 lb will impart an acceleration of 1 ft/s^2 to a mass of 1 slug [i. e., 1 lb = (1 slug) (1 ft/s^2)]. For dimensional homogeneity, it is clear that a slug = (lb•s^2)/ft. In the US Customary System, the gravitation constant g = 32.17 ft/s^2.

In addition to the quantities presented in Table 14.4, several other quantities are useful for statics and strength of materials, listed in Table 14.5.

Table 14.5. Units of other frequently used quantities.

Unit of Measure	**US Customary**	**SI Equivalent**
Moment M	foot-pound (ft•lb)	newton-meter (N•m)
Stress σ	pound/square foot (lb/ft^2)	Pascal (Pa = (N/m^2)
Strain ε	dimensionless	dimensionless

The quantities in Table 14.5 are given in terms of the basic units for length, time and force. However, in practice other units are often employed. For instance, moment may be expressed as in.•lb instead of ft•lb, and stress expressed as MPa (mega Pascal) instead of Pa (1 MPa = 1,000,000 Pa = 1 N/mm^2) or psi ($lb/in.^2$) instead of (lb/ft^2). Conversions between the two systems are listed in Table 14.6.

Table 14.6. Unit conversion factors.

Unit of Measure	**US Customary**	**SI Equivalent**
Acceleration	ft/s^2	0.3048 m/s^2
	$in./s^2$	0.0254 m/s^2
Area	ft^2	.0929 m^2
	$in.^2$	645.2 mm^2
Energy	ft•lb	1.356 J

Table 14.6. Unit conversion factors. (Continued)

Unit of Measure	US Customary	SI Equivalent
Force	Kip	4.448 kN
	lb	4.448 N
Impulse	lb•s	4.448 N•s
Length	ft	0.3048 m
	in.	25.40 mm
	mi	1.609 km
Mass	lb mass	0.4536 kg
	slug	14.59 kg
	ton mass	907.2 kg
Moment	ft•lb	1.356 N•m
	in.•lb	0.1130 N•m
Area Moment of Inertia	$in.^4$	$0.4162 \times 10^6\ mm^4$
Power	ft•lb/s	1.356 W
	hp	745.7 W
Stress and pressure	lb/ft^2	47.88 Pa
	$lb/in.^2$ (psi)	6.895 kPa
Velocity	ft/s	0.3048 m/s
	in./s	0.0254 m/s
	mi/h (mph)	0.4470 m/s
Volume	ft^3	$0.02832\ m^3$
	$in.^3$	$16.39\ cm^3$
	gal	3.785 l
Work	ft•lb	1.356 J

CONCLUSION

One of the characteristics of *engineering design*, as opposed to trial-and-error, is the use of *analysis*, which allows the engineer to make design decisions based on analysis, before parts are built. Engineering mechanics is an important analysis tool that can predict how physical parts of a design will react to forces. While a thorough understanding of this complex topic requires several semesters of in-depth study, the purpose of this chapter is to provide enough details to allow students to design parts that will neither break, nor bend too much.

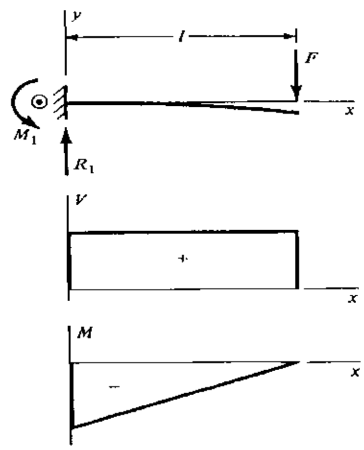
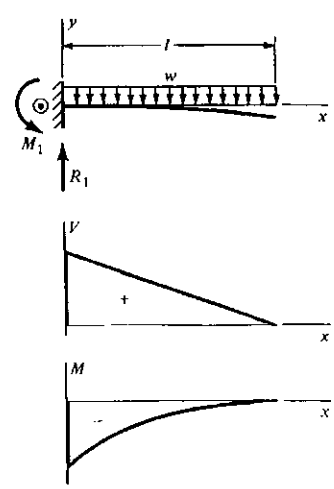
The following method applies to many design situations:

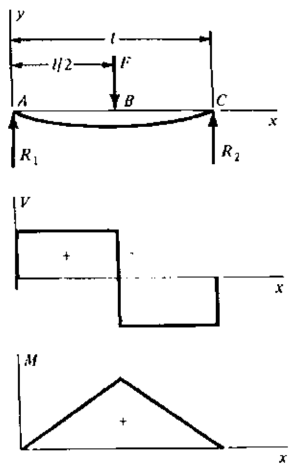
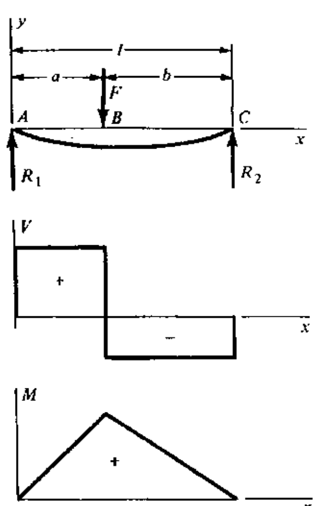
1. Use *statics* to understand the complete state of external and internal forces and moments that act on a body, assumed for the moment to be rigid.
2. Calculate the *stress* (e.g., Equations 14.3, 14.4) that develops inside a body when forces act on it.
3. Compare the stress to the strength of the material and calculate the factor of safety.
4. Calculate how much a body will *deform* in reaction to applied forces (e.g., Equations 14.6, 14.7).
5. Adjust the dimensions or material to make sure that the part will perform as intended.

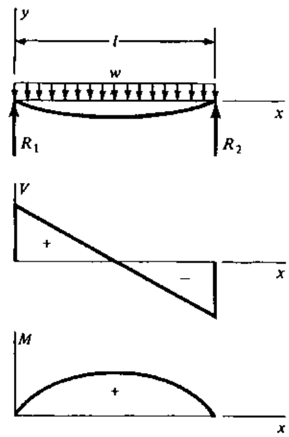
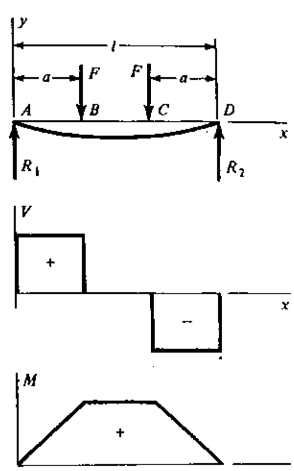
REFERENCES

1. Young, W.C., *Roark's Formulas for Stress and Strain,* Sixth Ed., McGraw-Hill, 1989.
2. Shigley, J.E. and Mischke, C.R., *Mechanical Engineering Design,* Fifth Edition, McGraw-Hill, 1989.
3. California Division of Industrial Safety, Department of Industrial Relations, Subchapter 6. *Elevator Safety Orders.*

APPENDIX 14.A: Shear, Moment and Deflection of Some Simple Loading of Beams.

a) Cantilever beam — concentrated load 	$R_1 = F$ $M_{MAX} = Fl$ $Y_{MAX} = -\frac{Fl^3}{3EI}$
b) Cantilever beam — uniform load 	$R_1 = wl$ $M_{MAX} = \frac{wl^2}{2}$ $Y_{MAX} = -\frac{wl^4}{8EI}$

c) Simple supports — center load 	$R_1 = R_2 = \frac{F}{2}$ $M_{MAX} = \frac{Fl}{2}$ $Y_{MAX} = -\frac{Fl^3}{48EI}$
d) Simple supports — intermediate load (a < b) 	$R_1 = \frac{Fb}{l} \quad R_2 = \frac{Fa}{l}$ $M_{MAX} = \frac{Fab}{1}$ $Y_{MAX} = -\frac{Fab(a^2 + b^2 - l^2)}{6EIl}$

e) Simple supports — uniform load 	$R_1 = R_2 = \frac{wl}{2}$ $M_{MAX} = \frac{wl^2}{8}$ $Y_{MAX} = -\frac{5wl^4}{384EI}$
f) Simple supports — axle loading 	$R_1 = R_2 = F$ $M_{MAX} = Fa$ $Y_{MAX} = -\frac{Fa}{24EI}(4a^2 - 3l^2)$