Calibration of Weirs

Introduction
A weir is a dam in an open channel. The flow over the weir can be determined by a single measurement: the upstream submergence or weir head. This is the distance between the weir crest (sharp edged) and the still water surface (the water surface before the drop-down curve). However, before the flow can be determined through the submergence, the weir must be calibrated. The calibration process entails establishing the discharge coefficient, \( C_d \), which adjusts the theoretical discharge equation to obtain the actual discharge.

The weirs used on the hydraulics bench for this experiment are known as sharp edge notches. They are made with sharp edges to reduce the amount of viscous friction in the fluid. However, in practice, the edge is actually manufactured with a small flat edge because the water would eventually erode the edge and have an adverse effect on weir performance. Another type of weir is a broad-crested weir. The small fountain in the south engineering courtyard is actually four broad-crested weirs. Unlike a sharp edged weir, a broad-crested weir is an elevated stretch of channel floor.

Objective
Calibrate two weirs by determining the discharge coefficient, \( C_d \).

Uses
With a calibrated weir, a single measurement of the weir head allows the determination of the flow rate in the channel.

Theory

Theoretical flow over a weir
Assume:
1. Bernoulli's assumption — streamlines are straight and parallel (no head loss)
2. The velocity distribution upstream from the weir is uniform.
3. The fluid particles move horizontally as they pass the weir crest.
4. The pressure in the nappe is atmospheric.
5. The influence of viscosity, turbulence and surface tension are negligible.

Using Bernoulli's equation, consider the motion of a particle of fluid flowing from 1 to 2 (see Figure).

\[
\frac{z_1}{2} + \frac{V_1^2}{2g} + \frac{P_1}{\gamma} = \frac{z_2}{2} + \frac{V_2^2}{2g} + \frac{P_2}{\gamma}
\]
From the diagram you can see that Bernoulli’s equation reduces to:

\[ H + \frac{V_1^2}{2 \cdot g} = (H-h) + \frac{V_2^2}{2 \cdot g} \quad V_2 = \sqrt{2 \cdot g \left( h + \frac{V_1^2}{2 \cdot g} \right)} \]

where:
- \( H \) = distance between the weir crest and the still water surface [L]
- \( h \) = distance of the water surface at point 2 below the still water surface [L]

We want to solve for the flow, \( Q \), over the weir crest. \( Q = \int_0^H (V_2 \cdot w) \, dz \),

where: \( w \) = width of rectangular weir [L]
For a rectangular weir, the width, is not a function of \( h \):

\[ Q = w \cdot \int_0^H \frac{2}{3} \cdot \sqrt{2 \cdot g} \cdot \left[ \left( H + \frac{V_1^2}{2 \cdot g} \right) - \left( \frac{V_1^2}{2 \cdot g} \right) \right] \, dh \]

This can be simplified by assuming the upstream velocity head is negligible compared to \( H \), \( H \gg \frac{V_1^2}{2 \cdot g} \). This assumption is less justifiable when the channel is narrow. As the cross sectional area decreases, the velocity increases, making the approach velocity much more significant. The equation for the theoretical flow rate over a rectangular weir is:

\[ Q_{\text{theory}} = \frac{2}{3} \cdot w \cdot \sqrt{2 \cdot g} \cdot H^2 \]

For a V-notch weir, \( w \) depends on depth:

\[ \tan \left( \frac{\theta}{2} \right) = \frac{w/2}{H-h} \quad w = 2 \cdot (H-h) \cdot \tan \left( \frac{\theta}{2} \right) \]

where: \( \theta \) = total angle of triangular weir
To find the flow rate:

\[ Q = \int_0^H (V_2 \cdot w) \, dz \]

Substituting in \( V_2 \) and \( w \), integrating and assuming \( V_1 << H \) gives the theoretical result for the flow over a V-notch weir:

\[ Q_{\text{theory}} = \frac{8}{15} \cdot \sqrt{2 \cdot g} \cdot \tan \left( \frac{\theta}{2} \right) \cdot H^\frac{5}{2} \]
The Coefficient of Discharge

Because real flows do not meet all the assumptions listed above, the actual flow will generally be less than the theoretically predicted flow. The coefficient of discharge, $C_d$, is an experimental correction factor which must be applied to the theoretical discharge value to obtain the actual discharge.

$$C_d = \frac{Q_{actual}}{Q_{theory}}$$

So how do you find $C_d$? Now that you know $Q_{theory}$ you can run an experiment where you measure $Q_{actual}$ for different values of $H$ ($H =$ upstream depth of water, where the weir crest is the datum).

1. If $C_d$ and $K$ are constant across all flow rates, it is simple to find $C_d$. A graph of $Q_{actual}$ against $H^{3/2}$ for a rectangular weir, or $H^{5/2}$ for a V-notch weir, should be a straight line having a slope equal to $C_d * K$. $Q_{actual}$ and $H$ are found by experiment and $K$ can be calculated.

Using the $C_d$ corrector, the actual flow rate is:

For a rectangular weir: $Q_{actual} = C_d \cdot K \cdot H^{3/2} \quad K = \frac{2}{3} \cdot w \cdot \sqrt{2 \cdot g}$

For a triangular weir: $Q_{actual} = C_d \cdot K \cdot H^{5/2} \quad K = \left(\frac{8}{15} \cdot \sqrt{2 \cdot g} \cdot \tan\left(\frac{\theta}{2}\right)\right)$

where:
- $Q_{act} =$ actual flow rate [L$^3$ T$^{-1}$]
- $Q_{intercept} =$ apparent flow rate from graph when $H = 0$ [L$^3$ T$^{-1}$]
- $C_d =$ coefficient of discharge [—]
- $K =$ meter constant

From dimensional analysis and experiments, the average value of $C_d$ for a rectangular weir is 0.622, and the value of $C_d$ for a V-notch weir 0.58 to 0.61.

If the $Q$ vs. $H^{x/2}$ graph appears to be a straight line, for practical purposes it is reasonable to assume that the relationship is linear and hence $C_d$ is constant. However, in cases where $C_d$ appears to be constant but does not have a zero intercept you need to take the intercept (when $H = 0$) into account.

$$Q_{actual} = C_d \cdot K \cdot (H^{3/2} \text{ or } H^{5/2}) + Q_{intercept}$$

2. If $C_d$ is not a constant, try to find a functional relationship:

$$Q_{actual} = a \cdot H^n$$

Where $a$ and $n$ are constants (you can use a power fit or trend line in Excel)


**Lab**

**Equipment**
hydraulics bench, stopwatch, weirs module (delivery nozzle, stilling baffle, rectangular and V-notch weirs), hook and point gauge, ruler or tape measure.

**Procedure**
1. Before turning the bench pump on make sure the outlet is covered with the delivery nozzle (and the quick release is locked) to prevent a geyser of water from flowing up to the lab ceiling when the pump is turned on.
2. Slide the stilling baffle into place and secure the first weir plate to the end of the channel with the thumb nuts. Place the instrument carrier containing the hook and point gauge over the channel.
3. Measure the top width of the notch.
4. Zeroing the Vernier scale
   a) Take the fine adjustment nut, at the top of the mast, and screw it so that half of the threads are above the nut and half are below.
   b) Set the Vernier gauge to datum with the point on the bottom of the weir. Slide the point in the mast until the point rests at the top of the weir crest.
   c) Lock the locking screw at the top of the mast to keep the point at that level.
   d) Slide the Vernier scale on the mast to line up the zeros of both the Vernier and the mast scale. Now, the point can be moved up using the mast and the fine adjustment nut.
   e) Now the scale should be set up correctly for this weir. Use the mast and the fine adjustment nut to move the point to the surface of the water. When measuring the height of the water above the weir, the point should just barely touch the water's surface. This can be easily seen if viewed from the side at a level close to the water's surface.
5. Open the flow valve a little before turning on the pump.
6. Start with the highest flow rate possible without overflowing the channel.
7. After the flow and surface in the channel stabilize, use the timed volume method to find the flow rate. Record the head of height of the still water surface above the notch, H.
8. Decrease the flow rate after each volume and water height measurement is taken. Adjust the flow control knob to obtain heads decreasing in steps equal intervals so that the last measurement is just above the notch (5-10 mm). Decreasing the flow rate instead of increasing the flow rate will speed up the stabilization of the water surface between measurements.
9. Repeat steps 7 and 8. You must take 9-10 measurement for the rectangular weir, and 7-8 measurement for the triangular weir.
10. Repeat steps 2-9 for the second weir.
Results

Rectangular: \( w = 2 \) in
V-notch: \( \theta = 90 \) degrees

For Each Weir:
1. Plot \( Q \) versus \( H^{3/2} \) or \( H^{5/2} \) and determine \( C_d \) and \( Q_{\text{intercept}} \) from the graph.
2. For each data point, use the appropriate \( Q_{\text{actual}} \) equation above to find \( C_d \). Plot \( C_d \) vs. \( H \).
3. Is \( C_d \) constant for this notch?
4. Using a power fit (use a spreadsheet), determine an empirical formula describing the \( Q-H \) relationship such that \( Q = a \cdot H^n \).
5. How do \( C_d \) and \( n \) (NOTE: \( a = C_d \cdot K \)) compare to theory?
6. Estimate an average value of \( C_d \) for the range of the test and compare it to theory.

Table 1: Data

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